Forecasting economic time series using score-driven models with mixed-data sampling

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Outline

Introduction

GAS models

- The class of **Generalized Autoregressive Score** (GAS) models of Creal et al. (2013) and Harvey (2013) is

$$y_t \sim p(y_t|f_t;\theta),$$

$$f_{t+1} = \delta + \phi f_t + \alpha s_t,$$

where $p(\cdot|f_t;\theta)$ is the conditional density of y_t given f_t.

- The peculiarity of GAS models is given by the **score innovation** s_t

$$s_t = S_t u_t, \quad u_t = \frac{\partial \log p(y_t|f_t;\theta)}{\partial f_t},$$

where S_t is a scaling factor.

Example: GAS models

- Time-varying **mean:**

$$y_t = f_t + \epsilon_t,$$

  - If ϵ_t is **Normal**: ARMA(1,1) model
  - If ϵ_t is **Student-t**: Robust filter, *Harvey and Luati (2014)*

- Time-varying **variance:**

$$y_t = \sqrt{f_t}\epsilon_t,$$

  - If ϵ_t is **Normal**: GARCH(1,1) model
  - If ϵ_t is **Student-t**: Robust filter, *Creal et al. (2013)*

- In general, we can make time varying parameters of parametric distributions such as Poisson, Bernulli, ordered probit, etc.

Motivation for MIDAS-GAS

- GAS models have several appealing features:
  - **Flexible specifications**: Heteroscedasticity as well as heavy-tailed distributions can be easily accounted for.
  - **Easy to implement**: likelihood function in closed form.
  - **Powerful forecasting tools**: accurate predictions in applications.
- Often the variable we wish to forecast is measured with a **different frequency** than other explanatory variables. Appropriate methods are needed to account for this.
- The MIDAS-GAS filter exploits the full potential of GAS models and accounts for mixed frequencies through a MIDAS weighting scheme of the score-innovations.

Related literature

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- **Linear factor models**
  - Mariano and Murasawa (2003), Schumacher and Breitung (2008) and Blasques et al. (2016): Treating low frequency variables as missing values, Kalman filter is used to extract and forecast the low frequency signal.
  - Frale and Monteforte (2011): Factor model with high frequency variable transformed via MIDAS weighting scheme.
  - Marcellino and Schumacher (2010): 2 steps approach. First step, factors from high frequency variable obtained via PCA. Second step, MIDAS regression using the factors.
- **GAS models**
  - Creal et al. (2014): GAS filter where mixed frequencies are handled by introducing missing observations (Ad hoc method).

The MIDAS-GAS filter

Forecasting setting

- The aim is forecasting a variable of interest y_t^L, which is sampled at a **low frequency** L.
- An explanatory variable $\mathbf{x}_t^H = (x_{1,t}^H, \ldots, x_{n_x,t}^H)'$ that is sampled at an **high frequency** H is available. Here t denotes the time index of the low frequency variable.
- **Example:**
  - L is **quarterly** and H is **daily**
  - n_x is the numbers of days in a quarter (about 65)
  - $x_{i,t}^H$ is the explanatory variable the i-th day of quarter t

The MIDAS-GAS filter (i)

- **We consider the following structure:**
  - Dynamic properties of y_t^L and $\mathbf{x}_t^H$ depend on a common time-varying parameter f_t.
  - y_t^L and $\mathbf{x}_t^H$ are independent conditional on f_t.

- Under these conditions, the classic **score innovation** for f_t is

$$s_t = s_t^y + \sum_{i=1}^{n_x} s_t^{x_i},$$

where s_t^y is the score contribution from y_t^L and $s_t^{x_i}$ from $x_{i,t}^H$.

- **Idea:** transform score contribution through a MIDAS weighting scheme to account for time order of the scores $s_t^{x_i}$, $i = 1, \ldots, n_x$.

The MIDAS-GAS filter (ii)

- The **MIDAS-GAS model** is specified as

$$y_t^L \sim p_y(y_t^L | f_t; \theta), \quad x_{i,t}^H \sim p_x(x_{i,t}^H | f_t; \theta), \ i = 1, \dots, n_x,$$

where

$$f_{t+1} = \delta + \phi f_t + \alpha_y s_t^y + \alpha_x \sum_{i=1}^{n_x} \omega_i(\boldsymbol{\varphi}) s_t^{x_i},$$

where $\omega_i(\boldsymbol{\varphi})$, $i = 1, \dots, n_x$, represent the MIDAS weights.

- The **exponential Almon Lag weights** are

$$\omega_i(\boldsymbol{\varphi}) = \frac{\exp(\varphi_1 i + \varphi_2 i^2)}{\sum_{i=1}^{n_x} \exp(\varphi_1 i + \varphi_2 i^2)},$$

where $\boldsymbol{\varphi} = (\varphi_1, \varphi_2)'$.

Weighted likelihood estimation

- The objective of the MIDAS-GAS model is forecasting the low frequency variable y_t^H. We consider the **weighted likelihood approach of Blasques et al. (2016)** to account for this.

- Estimation of the static parameter vector θ is performed by maximizing the **weighted likelihood**

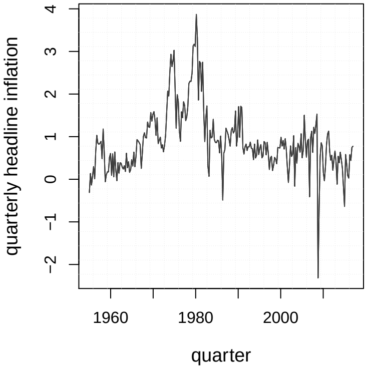
$$L_T^W(\theta) = \sum_{t=1}^{T} \log p_y(y_t^L | f_t; \theta) + W \sum_{t=1}^{T} \sum_{i=1}^{n_x} \log p_x(x_{i,t}^H | f_t; \theta),$$

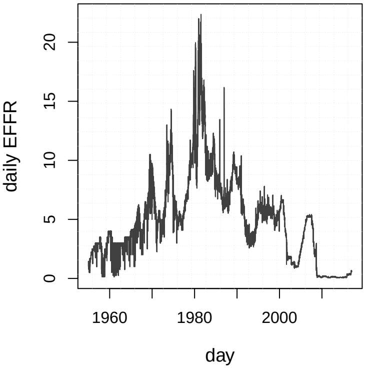
  where T is the sample size and $W \in [0,1]$ is a given weight.

- We set $W = 0$: **consider only the likelihood contribution of y_t^H** (identification: parameters of x_t^H enter the likelihood through f_t).

Forecasting Inflation with MIDAS-GAS

Our aim is forecasting **quarterly headline inflation** using **daily effective federal funds rates** (EFFR) as predictor (Armesto et al. 2010). We consider data from 1955 to 2016.





MIDAS-GAS factor model

- **MIDAS-GAS factor model** with conditional heteroscedasticity:

$$\begin{bmatrix} y_t^L \\ \boldsymbol{x}_t^H \end{bmatrix} = \begin{bmatrix} 1 \\ \lambda_\mu^x \mathbf{1}_{n_x} \end{bmatrix} \mu_t + \sigma_t \begin{bmatrix} \epsilon_{y,t} \\ \boldsymbol{\epsilon}_{x,t} \end{bmatrix},$$

  where $\epsilon_{y,t}$ and $\boldsymbol{\epsilon}_{x,t}$ are independent errors.

- The **time-varying** factor **mean** and **variance** are:

$$\mu_{t+1} = \mu_t + \alpha_\mu^y s_{\mu,t}^y + \alpha_\mu^x \sum_{i=1}^{n_x} \omega_i(\varphi) s_{\mu,t}^{x_i},$$

$$\sigma_{t+1}^2 = \delta + \phi\sigma_t^2 + \alpha_\sigma^y s_{\sigma,t}^y + \alpha_\sigma^x \sum_{i=1}^{n_x} \omega_i(\varphi) s_{\sigma,t}^{x_i},$$

  The score innovations depend on the distribution of the errors.

Score innovations

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We consider two different specifications for the error terms

- **Normal distribution** ($\epsilon_{y,t} \sim N(0,1)$ and $\epsilon_{x_i,t} \sim N(0,1)$):

$$s^{y}_{\mu,t} = y^L_t - \mu_t, \qquad s^{x_i}_{\mu,t} = x^H_{i,t} - \lambda^x_\mu \mu_t,$$

$$s^{y}_{\sigma,t} = (y^L_t - \mu_t)^2 - \sigma^2_t, \qquad s^{x_i}_{\sigma,t} = (x^H_{i,t} - \lambda^x_\mu \mu_t)^2 - \sigma^2_t.$$

- **Student-t distribution** ($\epsilon_{y,t} \sim t_\nu(0,1)$ and $\epsilon_{x_i,t} \sim N(0,1)$):

$$s^{y}_{\mu,t} = \frac{(\nu+1)(y^L_t - \mu_t)}{(\nu-2) + (y^L_t - \mu_t)^2 \sigma_t^{-2}}, \qquad s^{x_i}_{\mu,t} = x^H_{i,t} - \lambda^x_\mu \mu_t,$$

$$s^{y}_{\sigma,t} = \frac{(\nu+1)(y^L_t - \mu_t)^2}{(\nu-2) + (y^L_t - \mu_t)^2 \sigma_t^{-2}} - \sigma^2_t, \qquad s^{x_i}_{\sigma,t} = (x^H_{i,t} - \lambda^x_\mu \mu_t)^2 - \sigma^2_t.$$

The Student-t distribution leads to a robust update

Estimated models

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	ν	λ_μ^x	α_μ^y	α_μ^x	α_σ^y	α_σ^x	δ	ϕ	llik
t-MIDAS-GASg	7.36	1.49	0.46	0.14	0.23	0.07	2.21	0.87	-319.83
t-MIDAS-GAS	5.16	1.38	0.45	0.13	-	-	2.87	-	-333.89
MIDAS-GASg	-	1.62	0.54	0.11	0.23	0.21	2.31	0.81	-476.68
MIDAS-GAS	-	1.61	0.51	0.09	-	-	3.44	-	-511.36

Table: Parameter estimates of the models.

Estimated MIDAS weights

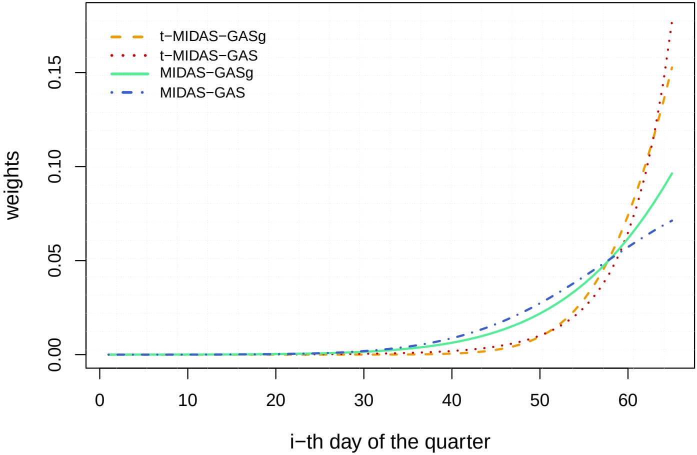

Figure: Estimated MIDAS weighting function.

Out-of-sample experiment

- We consider out-of-sample period from 1993 to 2016. Forecasts are based on rolling window approach.

- Besides our **MIDAS-GAS** models, several competing models are included in the comparison: **MIDAS regression**, **factor-MIDAS** (Frale and Monteforte, 2011), **AR** and **standard GAS** models.

- We evaluate forecasting accuracy using the following criteria:
  - **Point forecasts:** Mean Squared Error
  - **Density forecasts:** log score criterion

Point forecasts

	Mean squared error ratio					
	$h=1$	$h=2$	$h=3$	$h=4$	$h=5$	$h=6$
t-MIDAS-GASg	1.00	1.00	**1.00**	1.00	1.00	1.00
t-MIDAS-GAS	1.03	1.00	1.02	1.04	1.05	1.02
MIDAS-GASg	1.02	**0.99**	1.01	**0.95**	**0.96**	**0.95**
MIDAS-GAS	1.00	1.00	1.07	1.03	1.01	1.01
t-MIDAS(2)	0.99	1.13	1.14	1.07	1.11	1.15
t-MIDASg(2)	**0.96**	1.05	1.10	1.02	1.04	1.09
MIDAS(2)	0.99	1.13	1.13	1.06	1.10	1.13
MIDASg(2)	0.98	1.12	1.13	1.04	1.06	1.09
t-AR(2)	1.03	1.12	1.13	1.03	1.16	1.14
t-ARg(2)	1.06	1.09	1.10	1.03	1.14	1.14
AR(2)	1.00	1.10	1.09	1.02	1.12	1.14
ARg(2)	0.98	1.09	1.08	1.00	1.12	1.13
fMIDAS	1.00	1.11	1.07	1.03	1.01	1.01

Table: The benchmark model is t-MIDAS-GASg.

Density forecasts

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	Log score criterion					
	$h = 1$	$h = 2$	$h = 3$	$h = 4$	$h = 5$	$h = 6$
t-MIDAS-GASg	-1.422	**-1.546**	**-1.482**	**-1.612**	**-1.593**	**-1.579**
t-MIDAS-GAS	**-1.411**	-1.563	-1.548	-1.636	-1.620	-1.669
MIDAS-GASg	-2.230	-2.185	-2.181	-2.196	-2.195	-2.120
MIDAS-GAS	-2.218	-2.260	-2.212	-2.242	-2.222	-2.208
t-MIDAS(2)	-1.760	-1.859	-2.016	-2.041	-1.983	-1.861
t-MIDASg(2)	-1.977	-2.118	-1.815	-1.940	-2.095	-1.837
MIDAS(2)	-2.301	-2.278	-2.340	-2.321	-2.312	-2.321
MIDASg(2)	-2.219	-2.246	-2.254	-2.248	-2.245	-2.260
t-AR(2)	-1.766	-2.128	-2.199	-1.939	-1.884	-2.325
t-ARg(2)	-1.777	-1.946	-2.121	-1.877	-1.829	-1.899
AR(2)	-2.281	-2.241	-2.356	-2.433	-2.257	-2.224
ARg(2)	-2.174	-2.180	-2.263	-2.409	-2.160	-2.183
fMIDAS	-2.217	-2.260	-2.312	-2.300	-2.282	-2.297

Conclusion

- We have introduced a **novel GAS filter with MIDAS weighting scheme** for forecasting economic variables.
- The proposed approach is **easy-to-implement and very flexible**. It can account for heavy tails as well as heteroscedasticity.
- **Forecasting results are promising**. The MIDAS-GAS filter outperforms standard competing models in forecasting quarterly headline inflation using daily federal funds rates.

Main references

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