Accelerating Score-Driven Models:
Optimality, Estimation and Forecasting

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1. Background

- The Generalized Autoregressive Score (GAS) framework of Creal *et al.* (2013) and Harvey (2013) provides a general method to specify time varying parameter models.
- Time series often shows changing dynamics. We can have high persistence in some time periods and low persistence in others. GAS models may not be always able to properly handle these situations.
- We propose an extension to GAS models that allows to capture more complex dynamic behaviors.

2. Proposed approach

- Consider the following GAS model for time varying mean

$$y_t = \mu_t + \epsilon_t, \quad \epsilon_t \overset{\text{iid}}{\sim} N(0,\sigma^2),$$
$$\mu_{t+1} = \mu_t + \alpha(y_t - \mu_t).$$

The parameter α is particularly relevant as it determines the speed of adjustment of μ_t when a new observation becomes available.

- We introduce time variation in α. The accelerating GAS (aGAS) model is described by the following equations

$$\mu_{t+1} = \mu_t + \alpha_{t+1}(y_t - \mu_t), \quad \alpha_{t+1} = \exp(f_{t+1}),$$
$$f_{t+1} = \omega_\alpha + \beta_\alpha f_t + \alpha_\alpha (y_t - \mu_t)(y_{t-1} - \mu_{t-1}).$$

The parameter α_t is driven by products of past innovations.

- The aGAS model can be useful in situations where the amount of local information is changing over time.

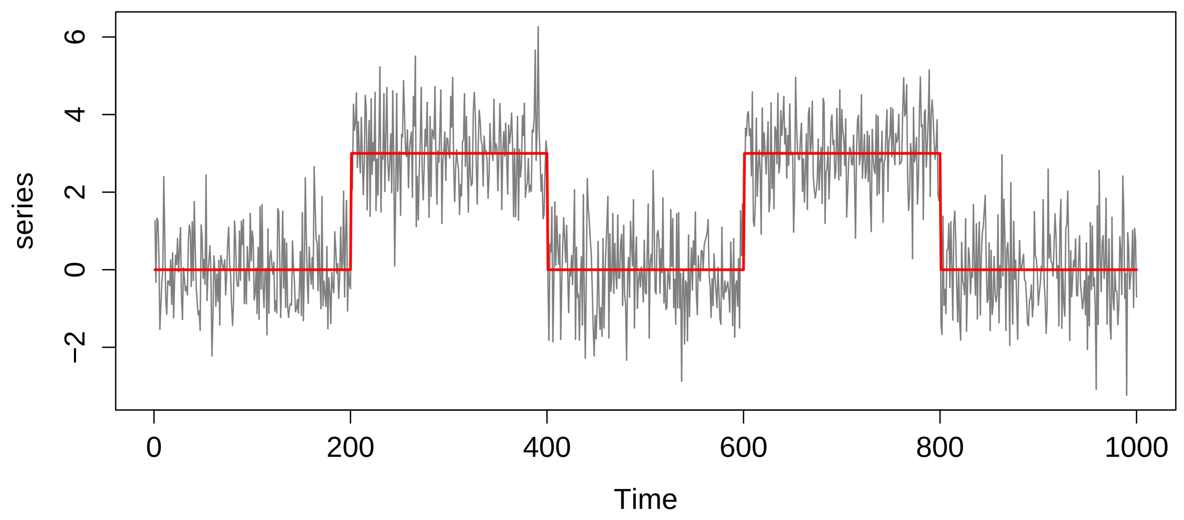

Figure 1: Simulated series. The red lines denotes the mean of the series.

We would like to update quickly the mean after the breaks and maintain it constant when the level is not changing.

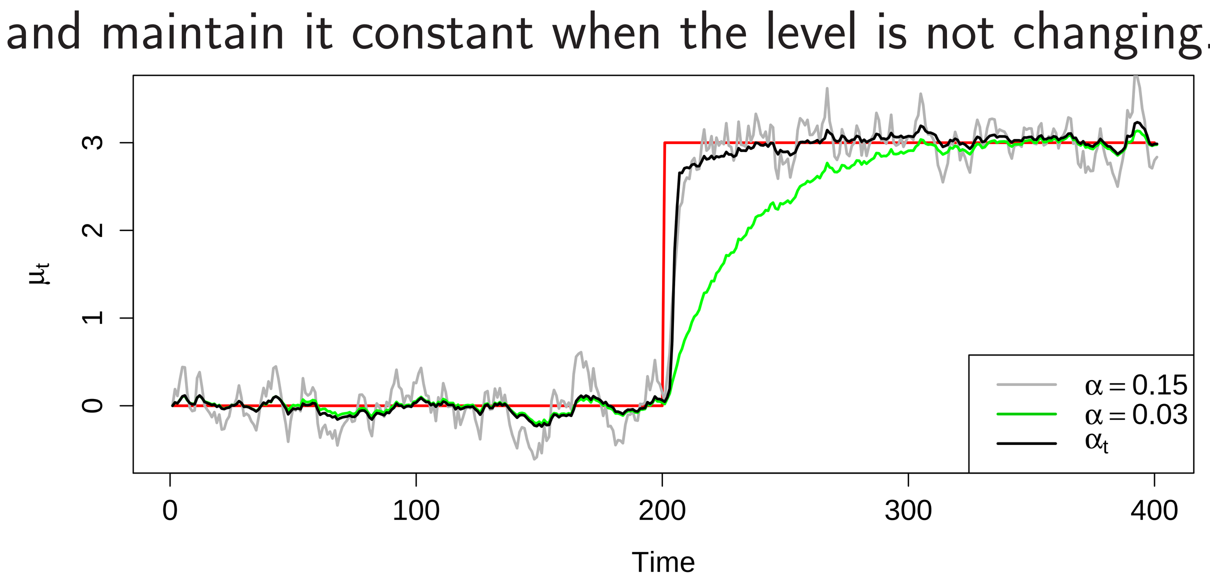

Figure 2: Filtered μ_t for different values of α.

Figure 2 shows that with a constant α there is a trade-off between updating quickly the mean and being exposed to the noise. Whereas, using the dynamic α_t can allow to update quickly μ_t only after the brake in the level.

- The model presented here is a specific case of GAS model for time varying mean and Gaussian errors. Our approach can be applied in a general GAS setting by replacing the innovation $y_t - \mu_t$ with the score of the predictive log-likelihood.

References

Blasques, F., Koopman, S. J. and Lucas, A. (2015). Information-theoretic optimality of observation-driven time series models for continuous responses. *Biometrika* **102**, 325–343.

Creal, D., Koopman, S. J. & Lucas, A. (2013). Generalized Autoregressive Score models with applications. *Journal of Applied Econometrics* **28**, 1099–1255.

Harvey, A. (2013). Dynamic Models for Volatility and Heavy Tails: With Applications to Financial and Economic Time Series, Econometric Society Monograph. *New York: Cambridge University Press.*

3. Simulation experiment

- We generate time series from the following Data Generating Process

$$y_t = \mu_t^o + \eta_t, \quad \eta_t \overset{\text{iid}}{\sim} N(0,1),$$

where μ_t^o is given by

$$\mu_t^o = \begin{cases} 0 & \text{if} \quad \sin\left(\gamma^{-1}10^{-2}(\pi t - 1)\right) \geq 0 \\ \delta & \text{if} \quad \sin\left(\gamma^{-1}10^{-2}(\pi t - 1)\right) < 0. \end{cases}$$

The series in Figure 1 is a realization from this process with $\delta = 3$ and $\gamma = 2$.

- The objective is comparing the ability to approximate μ_t^o of the GAS and the aGAS model

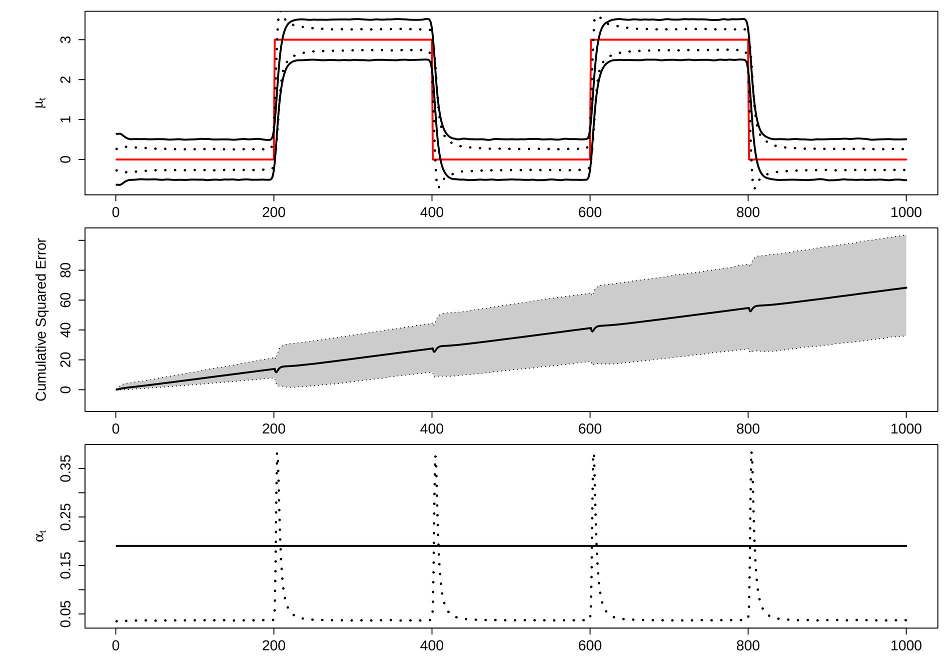

Figure 3: 95% confidence bounds, MSE and filtered α_t.

	$\gamma = 1.0$		$\gamma = 1.5$		$\gamma = 2.0$		$\gamma = 2.5$	
	GAS	aGAS	GAS	aGAS	GAS	aGAS	GAS	aGAS
$\delta = 0.0$	**3.86**	3.99	**3.86**	3.99	**3.86**	3.99	**3.86**	**3.99**
$\delta = 0.5$	22.34	**22.33**	20.19	**20.19**	18.17	**18.13**	17.05	**16.94**
$\delta = 1.0$	31.69	**31.40**	28.57	**28.07**	25.70	**24.91**	23.99	**22.89**
$\delta = 1.5$	39.21	**38.13**	35.25	**33.56**	31.66	**29.14**	29.48	**26.31**
$\delta = 2.0$	45.78	**43.50**	41.05	**37.62**	36.81	**31.97**	34.21	**28.47**
$\delta = 2.5$	51.78	**48.29**	46.30	**41.26**	41.45	**34.64**	38.47	**30.60**
$\delta = 3.0$	57.38	**53.09**	51.18	**45.02**	45.75	**37.58**	42.40	**32.83**
$\delta = 3.5$	62.71	**58.05**	55.80	**48.98**	49.79	**40.91**	46.08	**35.54**

Table 1: MSE between the true μ_t^o and the filtered μ_t.

- **Summary:**
  1. The dynamic α_t allows to update quickly μ_t only after the breaks.
  2. The aGAS is more robust against the noise component η_t when μ_t^o is constant.
  3. The aGAS model outperforms the GAS model in terms of MSE.

4. Empirical application: US consumer price inflation

- The series exhibits rapid changes in the level during the 80s, whereas, the level changes slowly in other periods.

Figure 4: Quarterly US consumer price inflation series.

- An aGAS model with Student-t error distribution and time varying variance is estimated.

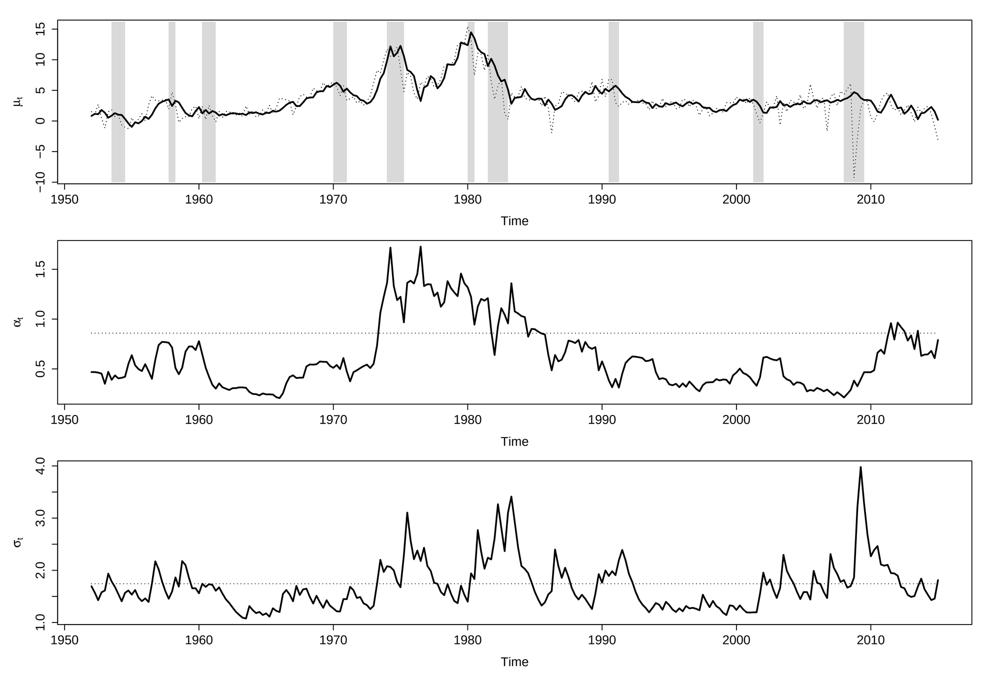

Figure 5: 1st plot: μ_t. 2nd plot: α_t. 3rd plot: σ_t^2.

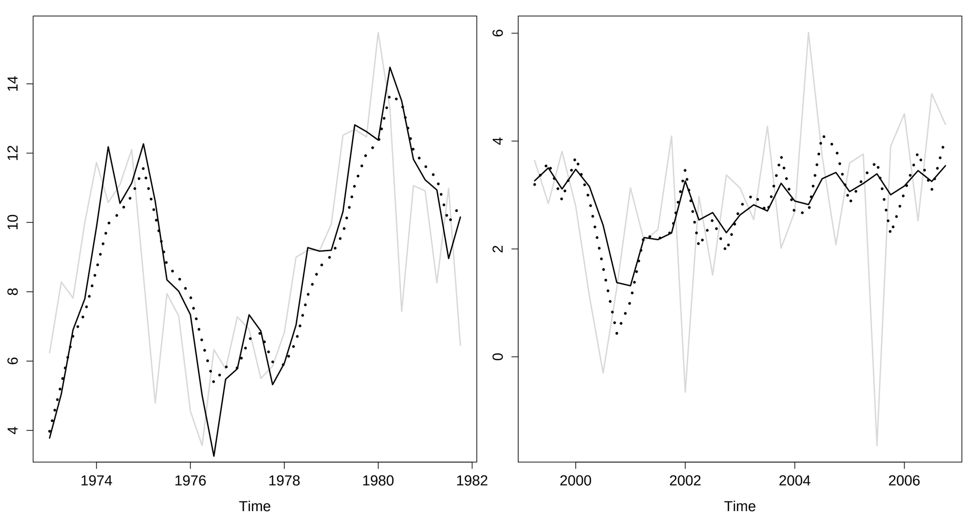

Figure 6: Dashed line: GAS. Continuous line: aGAS.

- **Summary:**
  1. The filtered α_t is particularly high in the 80s.
  2. The aGAS mean is updated quickly in the 80s and slowly in other periods.
  3. The aGAS model outperforms the GAS model both in-sample and out-of-sample.

5. Final remarks and conclusion

- The considered aGAS specification for the time varying parameter is justified by an optimality argument in the spirit of Blasques *et al.* (2015).
- The aGAS approach can be applied to a more general class of models such as GARCH-type models.
- Empirical applications and the simulation experiment show how our approach can be useful in practice and enhance the performances of score-driven models.